Theories of location*

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1 Introduction

Metaphysicians of space and time are fond of talking about objects being present at, wholly present at, or existing at certain times, or occupying certain regions of space, or even regions of space-time. Take, for example, this famous set of definitions due to Mark Johnston and David Lewis:

> Let us say that something *persists*, iff, somehow or other, it exists at various times; this is the neutral word. Something *perdures* iff it persists by having different temporal parts, or stages, at different times, though no one part of it is wholly present at more than one time; whereas it *endures* iff it persists by being wholly present at more than one time. (Lewis 1986, p. 202)

A great deal of debate has been conducted in this terminology: debates about whether anything does endure or perdure; about the ontology of temporal parts; about whether it makes sense to apply this kind of thinking to space, as well as to time (we can ask, for example, the analogous questions whether things are extended by being entended, or pertended); about whether it can be applied to space-time, and if so, to relativistic space-time. These debates have been fruitful, but cursed with a certain amount of imprecision. People sometimes talk past each

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other because they mean subtly different things by “is present at”, “occupied”, and the like, or because they are simply confused about what they do mean.

In this paper, I aim to clear up this problem of imprecision. I put forward a logical framework for discussing metaphysical debates that are couched in terms of what I call *location relations* — “present at”, “occupies”, “exists at” and so on. This framework consists of a formal system the theorems of which might be imagined to be all the conceptual truths about location relations (or perhaps even just all the truths of which it is not in dispute whether they are conceptual truths). I can then formulate, for example, the thesis that everything perdures as a sentence of my formal system, and see, in a formally rigorous way, what its consequences are. I can also show that we have a free choice of primitive location relations, and prove that certain location relations are inter-definable.

I describe my formal system as a theory of location because it is a theory in the logical sense — it is a set of axioms and definitions in the language of first order predicate calculus. But it isn’t a theory in the sense of a complete metaphysical account of location. My goal is to construct a minimal background within which such complete accounts can be discussed.

It might be useful to contrast what I am doing here with some similar projects. I am not the first person to state a theory of location. Casati and Varzi do so in their (1999). Their system however, is much stronger than mine, and is not intended to be neutral with respect to metaphysical disputes. Casati and Varzi also do not put their system to temporal or spatio-temporal uses. I discuss their theory of location further in section 7.

Many other people have attempted to regiment the Johnston/Lewis definitions of perdurance and endurance, or to define concepts like “temporal part”. A good example can be found in Sider (2001). Sider actually has two definitions of “temporal part”, and the difference between them is important. Sider’s first definition, in terms of time-indexed mereology, is designed to appeal to his opponents; his second definition, in terms of atemporal mereology is designed to express the theory he actually believes. It is this latter definition that is close to what I am attempting. Sider’s definitions, however, both fall short of the generality I aspire to by assuming that if objects have proper temporal parts at all, then they have instantaneous ones. In section 5 I show how to define “temporal part” in a way that does not assume this.

Finally, Gilmore (2006) gives a theory of location, informally, and with a very different structure from mine. He also defines “endurance” in a very different way from the way I would (and, I think, different from the Johnston/Lewis definition quoted above). I briefly say why I disagree with his treatment of the topic in section 5.

2 Location

Before we can think about temporal parts, we need to clarify what is meant by "exists at", or as I will say "is located at". This relation is not conceptually confined to times. We can also think of objects existing in places, or even regions of space-time. The formal structure I will describe in this section is the same in each case. Indeed, it will be easiest if we begin by thinking about the spatial case.

Let us say that I am *weakly located* in my office iff I am in my office in the weakest possible sense: iff my office is not completely free of me.[1] I should count as weakly located in my office when I am sitting at my desk, when I am reaching an arm out of the window, or when I am reaching an arm in the window from the street outside. Let us say that I am *entirely located* in my office iff I am in my office and I am not anywhere outside my office; that is, iff I am in my office and everywhere outside my office is completely free of me. I am entirely in my office when I am sitting at the desk, but not when I am reaching an arm in or out of my window. Let us say that I pervade, or am *pervasively located* in, any place none of which is free of me. I don't ever pervade my office, but I do pervade the region exactly occupied by my left big toe. Finally, I am *exactly located* anywhere that I am both entirely and pervasively located. My exact location is like my shadow in substantival space.

These concepts are interrelated in interesting, formally expressible ways. To do that we need a way of formally expressing them:

$x@_{\circ}r$	for "x is weakly located at r"
$x@_{<}r$	for "x is entirely located at r"
$x@_{>}r$	for "x pervades r"
$x@r$	for "x is exactly located at r"

And we also need a way of expressing the relationships between the regions at which things are located: the relationship of one region being a *subregion* of

[1] In these examples, I am treating "my office" as the name of a *place*, not of a material thing. Or, speaking more strictly, I am assuming that "my office" names a place where things might be located; I am neutral as to whether that place is identical to some material things. This type of issue is discussed further in section 8.

another (as for example, my office is a subregion of the university campus). The subregion relation is intended to be reflexive — each region counts as its own subregion. A subregion of r that is not r itself is called a *proper subregion* of r. Another important relation between regions is that of *overlap*, or having a common subregion (as Australasia and Polynesia overlap on New Zealand):

$r < s$	for "r is a subregion of s"
$r \ll s$	for "r is a proper subregion of s"
$r \circ s$	for "r overlaps s"
$r \wr s$	for "r is disjoint from (i.e. does not overlap) s"

The four location relations given above can be inter-defined. An elegant way of thinking about them is to start with the idea of exact location, and then define the others in the following way: an object is entirely located in every super-region of its exact location; it pervades every subregion of its exact location; and it is weakly located at every region overlapping its exact location. This is shown formally in the definitions below (and I have chosen the notation for my location relations to remind us of these definitions):

(1) $$x@_{<}r \equiv_{df} (\exists s)(x@s \wedge s < r)$$

(2) $$x@_{>}r \equiv_{df} (\exists s)(x@s \wedge r < s)$$

(3) $$x@_{\circ}r \equiv_{df} (\exists s)(x@s \wedge r \circ s)$$

It's important, however, that we don't have to count exact location as a primitive of our theory. We could instead start with the more everyday concept of weak location — the "in" of "I am in my office".[2]

[2] A number of people objected to this. Am I really in my office if most of me is in the corridor, and just the tip of my toe is over the threshold? Well, suppose I arrive in the department and rush up to my office and stand in the corridor in just that way. At the same time a student asks the department secretary whether I am in my office. The secretary is in a position to know where I am. What should the secretary say?

It would not be crazy for the secretary to say "yes" — that shows that it is not crazy to think that I am in my office when just a little bit of me is. Of course there are also cases where it would be misleading for the secretary to say "yes" even though I am weakly located in my office. There must be a lot of pragmatic contextual goings on here — my only claim here is that weak location is closer to the everyday sense of "in" than exact location.

The definitions in that case are those suggested by the glosses I gave when I introduced the terms:

(4) $$x@_{<}r \equiv_{df} x@_{\circ}r \wedge (\forall s)(x@_{\circ}s \rightarrow r \circ s)$$

(5) $$x@_{>}r \equiv_{df} (\forall s)(r \circ s \rightarrow x@_{\circ}s)$$

(6) $$x@r \equiv_{df} (\forall s)(r \circ s \leftrightarrow x@_{\circ}s)$$

The difference between (4) and (5) is noteworthy. Why does (4) have that extra first conjunct? If it were gone, then the definition of $@_{<}$ would just be the converse of the definition of $@_{>}$, which would seem more elegant. The reason is that I am not assuming that everything is in space. Suppose that the number seven is not in space. Then $(\forall s)(7@_{\circ}s \rightarrow r \circ s)$ is true, because the antecedent of the conditional is always false. But, in the sense of "entirely located" that I am trying to capture, the number seven is not entirely located everywhere; we need the extra conjunct to ensure that our definition of entire location agrees. (5) and (6) don't suffer from this problem, because there the weak location predicate occurs in the consequent of the conditional. To put the point another way, the definitions (5) and (6) already guarantee that, if $x@_{>}r$ or $x@r$, then $x@_{\circ}r$, so they don't need it as an explicit extra conjunct.

There are two very general principles about location that are worth noting here; the principles of functionality, and of exactness:

(Functionality) $$(x@r \wedge x@s) \rightarrow r = s$$

(Exactness) $$(\exists r)(x@_{\circ}r) \rightarrow (\exists r)(x@r)$$

Functionality says that exact location is a function. Exactness says that everything that is anywhere has an exact location. Each of them can be made to follow from definitions. Functionality is a consequence of (6) (given some further premises about the behaviour of $\circ$), and exactness is a very obvious consequence of (3). Both sound extremely plausible, given what's meant by "exact location".

People sometimes have doubts about Exactness. One reason for this doubt concerns objects part of which are in space, and part of which are not in space. Suppose that numbers are not in space, but that there are nonetheless "mixed" objects that have both numbers and concrete objects as parts. The mereological fusion of myself with the number seven (if there is such a fusion) would be such a mixed object.[3] In my view, this object is exactly located just where I am (this does not follow from what I have said so far, but it does follow from the additional metaphysical principles described in section 4). You might instead be tempted to say that this object is not exactly located anywhere, and indeed, that it is not entirely located anywhere, though it is weakly located everywhere that I am located.

I think that this kind of reasoning rests on a mistake about what "entirely located" means. Remember that for the mixed object to be entirely located somewhere is for there to be *no place else where it is*. The temptation to reject Exactness here is a temptation to treat objects not in space as if there were a special place, "not in space" in which those objects are spatially located. But that would be a mistake: an object not in space is not spatially located at all. (Later on, I'll draw a distinction between "entirely located" and "wholly located". Mixed objects won't count as *wholly* located anywhere).

Another reason for rejecting Exactness has to do with the possibility of objects that are more spatially fine-grained than space itself. Since this is a complicated topic, I discuss it in its own section, section 3. People also sometimes object to Functionality on the grounds that an enduring object might be exactly located at multiple times, or at multiple regions of space-time. That, objection, I think, rests on a mischaracterisation of endurance, and I discuss it in section 5.

So far I've been using spatial examples. The concepts I've been using can easily be extended to the temporal case. We can say of John Locke, for example, that he is weakly but not pervasively temporally located in the year 1704 (the year of his death); that he temporally pervades the year 1700; and that he is exactly temporally located in the interval between a certain instant in the year 1646 and a certain instant in the year 1704. For those who like space-time, we can also think of Locke having an exact location in space-time, which would overlap both Oxford and the year 1700.

Let me also make some comments about my usage of "time", "place", "region", and so on. I mean all of these very broadly. I'm not assuming that there are points or instants, but I do mean "time" and "place" in a sense according to which if there are points, then they are places; and if there are instants, then they are times. I also count intervals, and any scattered times as times, and I count both connected and scattered places as places, again if there are such things. I

[3] If you have trouble imagining that numbers enter into mereological relations at all, you might change the example of a mixed object, to, say, the fusion of the myself with the singleton set of the number seven. It's not too hard to think of sets as *parts* of their supersets. Of course the mixed object is still pretty weird, and you might doubt that there are such things. If there aren't, then the reason for doubting exactness that I am discussing is doomed to begin with.

don't assume that any particular place or time is divisible or that any particular set of places or times composes another (and for this reason I don't assume classical mereology); but I do stipulate that "is a place" and "is a time" are closed under division and composition. This is just what I mean by "place" and "time", not a deep metaphysical assumption.

3 Exactness in non-standard models of space

One way that Exactness might be imagined to fail is if there could be objects that were more spatially fine-grained than space itself. Suppose that space is gunky — that every region has a proper subregion — and that every region of space has a volume greater than zero. There are, in short, no points of space. But it could still be (it seems) that there are point-shaped things. A point-shaped thing would be entirely located in each of a series of spherical regions of arbitrarily small volume, but would not pervade any such region. No point-shaped thing can be exactly located in or pervade any non-point-like regions. So point-shaped things would be entirely (and this weakly) located in a number of regions, but not exactly located anywhere. Though such a theory of space might seem odd, it seems coherent, and this (says the objector) shows that Exactness is not a conceptual truth of location.[4]

An application of this kind of conception of space appears in Daniel Nolan's (2006) work on the Stoic conception of mixture. Chrysippus, apparently, believed that when wine and water are mixed, none of the wine is transformed into anything other than wine, and nor is any of the water transformed into anything other than water. Moreover, nor is a mixture of wine and water like a mixture of wheat and lentils, with small blobs of pure wine rubbing shoulders with small blobs of pure water (Nolan calls this a "juxtaposition"). Rather, the wine/water mixture is "mixed through and through" so that no portion of the mixture is pure wine, and no portion is pure water.

There has been much debate over whether this idea of "mixture through and through" or "blending" is coherent. The trouble is to say what has happened to the portions of water and wine that were mixed. Have they been destroyed or transformed into something other than pure water or pure wine? If so, then this is transformation, not blending. Are they still to be found, perhaps scattered,

[4]This type of objection to Exactness, involving point-shaped objects in gunky space, was given to me by Cody Gilmore; the following interpretation of Nolan, on which he denies Exactness, was suggested by Shieva Kleinschmidt.

but untransformed, in the mixture? If so, then the water/wine mixture is a mere juxtaposition — not really any different from a mixture of wheat and lentils.

Nolan offers a modern reconstruction that aims to save blending from incoherence, and I here reconstruct his reconstruction in terms of the theory of location. The Nolanian Stoic believes in a gunky space in which every region is continuous, and that water and wine are themselves made out of homogeneous gunky matter. A blend of water and wine is such that every subregion of the region in which the blend is exactly located contains some water and some wine (unlike a mixture of wheat and lentils, some subregions of the exact location of which contain nothing but lentil or nothing but wheat). That is to say, after the wine and water are mixed, the wine is entirely located in the exact location of the blend, and weakly located in every subregion of the exact location of the blend, but does not pervade any such region. It follows from this that after blending, the wine is weakly located in all the regions that the blend is weakly located in, but has no exact location. The wine is so scattered and discontinuous that it will not exactly fit any of the continuous regions of Nolanian space.

I have my doubts about whether this is really distinct enough from the wheat and lentils case to be what the Stoics were after. After all, on Nolan's view, there are still parts of the blend that are pure wine and pure water, rubbing shoulders in the blend just like grains of wheat and lentils. Unlike grains of wheat, they are not *exactly* located anywhere — but they are still parts of the blend, and they are still unmixed. Such an arcane metaphysical difference between blending and juxtaposition does not seem to do justice to the intuitive distinction between perfectly blended stuffs and jumbled piles of atoms. On the other hand, it may be that there is nothing better to answer to that distinction. Such an adept metaphysician and logician as Chrysippus would hardly believe in something that his contemporaries could easily tell was incoherent: that the blend has no part which is pure water, but yet the original pure water is part of it and has not changed its nature. Chrysippus *has to* agree that some parts of a blend are still pure, if he is not to be obviously contradicting himself; his view must therefore be that the pure parts of a blend are differently arranged from the pure parts of a juxtaposition. In any case, if Nolanian Stoic blends are coherent, Exactness is not a conceptual truth.

There's another way of generating counterexamples to Exactness in the same kind of spirit as those given above, but not involving gunky space. Someone might believe that space is so big that there is no maximal region. Let us call this conception of space, to coin a phrase, "knuggy space". Where the distinctive feature of gunky space is that every region of space has a proper subregion, the distinctive feature of knuggy space is that every region has a proper super-region.

For all I've said in this paper, space might be gunky, knuggy, both, or neither.[5] Just as no object can be exactly located at a point in gunky space (for there are none), no object can be exactly located at the maximal region in knuggy space (for there is none). If an object were located at the maximal region in a non-knuggy space, we might say that it was *omnipresent*. But, if you think about it, the theorist of knuggy space can (it seems) accommodate omnipresent objects, in a similar way to the way that the gunk theorist might accommodate point-shaped objects. An omnipresent object, we may say, pervades every region. If there is a maximal region, then it is entirely located there. If there is not, then it is not entirely located anywhere. In that latter case, we would have an object that is in space, but not exactly located anywhere.

These objections, I think, are the most serious problems with my framework so far. They may seem arcane, but I am making a very strong claim for Exactness — that it is a conceptual truth. It takes only the coherence of a counterexample to undermine that. What all three of these proposed counterexamples have in common is that they complicate the relationship between the geometrical properties of material things and the geometrical properties of space. They each also involve a counterexample to the schema "*x* is *S*-shaped iff the exact location of *x* is *S*-shaped". (Where *S* is "point", "discontinuous" and "omnipresent" respectively). I think that there is something bad about that — though I find it hard to put my finger on. The following dilemma is my best attempt to draw out the problem.

A substantivalist, I think, would not much like these proposals because they require that we make sense of the geometrical properties of material things independently of the geometrical properties of places. To say "there are things smaller than any region of space", is to treat space like a kind of ghostly matter that happens to inter-penetrate all material things, but which might or might not have parts that match the sizes and shapes of material things. This immediately raises the question of why we should believe in such spooky stuff, since it seems objects could have all sorts of geometrical properties whether or not there are places around to be those objects' exact locations. Every substantivalist should be asked why, on their view, space is not just a kind of ghostly and immovable matter, and I do not know what the answer should be — but I think that allowing for the possibility of objects whose shapes do not exactly match the shapes of any region of

[5] One technical difference between gunky and knuggy space is that gunky space could still be a model of Classical Extensional Mereology (CEM), for CEM does not say that there must be atoms. Knuggy space, however, cannot, since CEM does require that there be a universe — a fusion of everything. Knuggy space is, however, a model of Minimal Extensional Mereology (MEM), the mereological system I am employing in this paper.

space will make it harder to give one.

A relationalist about space, on the other hand, should also be dubious. Presumably the relationalist, at some point, is going to give a paraphrase of location talk into something relationalistically acceptable — some language not ontologically committed to regions of space. But why should the relationalist prefer to paraphrase one of these arcane, non-standard, gunky or knuggy theories of location when they could paraphrase a standard one instead? The non-standard models of space can only be attractive if they say something different from (and more correct than) the standard ones. So the relationalist can only be attracted to a non-standard model if she can say, in a relationalistically acceptable way, what it is getting right that the standard model is getting wrong. It is not at all clear what that could be.

4 Principles of partition

The location predicates we've seen so far, and the principles of functionality and exactness have to do with how physical objects stand to places and times, and to parts of those places and times. Metaphysical doctrines about temporal parts, however, require us to talk about the parts of those physical objects, not just of places and times themselves.

In my presentation, the mereological relation does double duty both as the relation between times (and places) and their subregions, and as the relation between material things and their parts — between humans beings and feet, between computers and keyboards. This is for formal convenience more than anything else — it would be a hassle to have to define mereological notions such as overlap and fusion twice over, once for times and places, and then again for material things. It's also fairly plausible as a metaphysical view that the places are *made out of* their subregions in the same sense that material things are made out of their parts. Should this seem objectionable, it is quite possible to remove this assumption while leaving intact everything else I say here. But it would be inconvenient, so I will leave the task to the reader who objects.

Using the formal apparatus of location relations and mereology, we can state some interesting and controversial metaphysical doctrines about parts and location. Consider, for example, the principle of arbitrary partition, which says that every object has a part exactly located in each region it pervades:

(Arbitrary partition) $$x@_{>}r \rightarrow (\exists y)(y < x \wedge y@r)$$

In the spatial case, this captures the idea that objects are divisible every way possible; every way corresponding to the ways space itself is divisible. There is such a thing (according to the principle) as the statue-shaped part of an uncut block of marble. For there is the region a statue would occupy, were one cut from the marble; and the marble pervades this region. So, by the principle, there is a part of the marble that is exactly located in that region. In the temporal case, this will turn out to be equivalent to the doctrine of perdurantism.

Arguments against this principle come in two varieties. There are arguments that purport to show that it leads to some absurd consequence, such as that no object can gain or lose parts. (van Inwagen 1981) There are also arguments that purport to show that it is empirically false. (Parsons 2000) I rather prefer the latter, but for the purposes of this paper, I am going to confine myself to the claim that the principle is not a conceptual truth about location.

I have little to say by way of argument here: simply that I can conceive of an object being extended without having any proper parts. Such an object would pervade many regions without having parts exactly located at those regions. For example, there might be a completely solid sphere that had no proper parts. The left half of the region this sphere exactly occupies is pervaded, but not itself exactly occupied, by the sphere; and since the sphere has no proper parts, it has no proper part that exactly occupies that region either. Since I can conceive of such things, my concepts cannot be getting in the way; therefore, it is not a conceptual truth that every region pervaded by an object has a part of that object exactly located in it. I'm not alone in this, either: there are a number of other philosophers who are in favour of extended simples.[6]

In thinking of such things, it's useful to make a distinction between being entirely located somewhere (in my terminology), and what is usually known as "being wholly located" somewhere. When I say that I am all in the office — that I am not stepping over the threshold or sticking an arm out of the window — there are two things I might mean. One is that I am *entirely* in the office, in the sense defined above: that everywhere disjoint from the office is free of me. The other thing I might mean is that I am *wholly* in the office: that none of me is missing from the office; every part of me is in the office.

It will be helpful to have a formal abbreviation for "wholly located":

[6] Among them are, John Bigelow (1995, pp. 21–27), Ned Markosian (1998), Fraser MacBride (1998, pp. 220–227), Peter van Inwagen (1981), and, according to van Inwagen, Aristotle (van Inwagen 1990, p. 98). Spinoza (1994, 1P12, 1P13) claims that the universe itself is an extended simple. Democritus is usually understood to have held that the atoms come in an infinite variety of shapes and sizes, which would also commit him to extended simples.

$x @_{\blacktriangleleft} r$ for "x is wholly located at r"

It can be defined in this way:

(7) $$x @_{\blacktriangleleft} r \equiv_{df} (\forall y)(y < x \rightarrow y @_{\circ} r)$$

Whole location and entire location tend to go around together. When I'm entirely in the office, I'm also wholly in the office; if I reach an arm out the window, I'm not entirely in the office, because I'm also in the street, but also, I'm not wholly in the office, because my arm isn't in the office. However, suppose the extended simple sphere were hovering over the sill of my window: it would be neither entirely in the office (for it is also in the street) nor entirely in the street (for it is also in the office). But it would be wholly in the office (for it has no part save itself, and that part is in the office) and wholly in the street (for it has no part save itself, and that part is in the street).

People sometimes give bad arguments for arbitrary partition by conflating "wholly located" with "entirely located". Of course, these labels I've chosen are technical — I don't claim that "wholly" and "entirely" have these different meanings in ordinary language. But it is one thing to say, "I have no parts missing from here", and another to say "No place disjoint from here is a place where I am". One statement asserts the non-existence of a certain kind of thing, the other the non-existence of a certain kind of place.

In the temporal case, the principle of arbitrary partition is the principle that objects have arbitrary temporal parts. People who believe the temporal version of the principle hold that persisting objects have proper parts exactly temporally located at each time they temporally pervade. For example, they hold that John Locke has a proper part that begins at the first instant of 1700, and ends at the last instant of that year. Those who are opposed to the temporal version of the principle hold that at least some things are wholly temporally located at some times at which they are not entirely temporally located. If Locke is such a thing, then he might be wholly temporally located in the year 1700, without being entirely temporally located in that year.

People who deny that persisting things have arbitrary temporal parts (or perhaps any proper temporal parts at all) are called *endurantists* and the things they believe in are said to *endure*, or to persist without having temporal parts — without *perduring*. Similarly, I say that some spatially extended things might have no arbitrary spatial parts (or perhaps no proper spatial parts at all). The things I am talking about, such as extended simples, may be said to be *entended*, or to be extended without having spatial parts — without being *pertended*.

Though I do not think that arbitrary partition principles are conceptually true, there are two related principles that seem as though they could be. Like arbitrary partition, they connect what sort of location an object has with what sorts of parts it has. The first of these is the principle of expansivity: the idea that an object cannot fail to be where its parts are. If part of me is in the office, then I can't be entirely absent from the office — my parts' exact locations must be subregions of my exact location.

There are a number of ways of stating this (another, weaker way is discussed in section 7). Here is one that I like:[7]

(Expansivity) $$x < y \wedge x@r \rightarrow (\exists s)(y@s \wedge r < s)$$

This principle is respected by every ordinary example of location that I can think of. It is also helpful in thinking about exotic cases. Consider the example I mentioned earlier, in relation to the principle of exactness. Suppose that the number seven is not in space, and that there is a mereological fusion of me with the number seven. It follows from expansivity that the fusion has an exact location which is a super-region of my exact location. Given that the fusion has an exact location at all, this seems right to me. However, it does not follow from Expansivity alone that the fusion's exact location is any particular super-region of my exact location.

It would follow from Arbitrary Partition and Expansivity that the fusion's exact location is identical to mine. Suppose, for reductio, that the fusion's exact location is a proper super-region of my exact location. Then there must be a subregion, r, of the fusion's exact location disjoint from my exact location, and pervaded by the fusion.[8] By Arbitrary Partition, then, the fusion has a part that is exactly located in r. But I am not located in r at all, so, by expansivity, I do not overlap anything exactly located at r. Nor does the number seven, for the same reason. But the fusion of me and the number seven by definition overlaps only things that overlap either me or seven. We have a contradiction; so the fusion's exact location is not a proper super-region of my exact location. Expansivity re-

[7]The label "expansivity" that I use refers to Goodman's (1977, p. 38) idea of an expansive predicate — a predicate satisfied by anything that has a part satisfying that predicate.

[8]This step employs the Strong Supplementation Principle discussed in section 6.

quires that the fusion's exact location be a super-region, proper, or improper, of mine so they must be identical.

5 Perdurance, endurance, and temporal parts

Consider again the Johnston/Lewis definitions of "persists", "endures", and "perdures":

> Let us say that something *persists*, iff, somehow or other, it exists at various times; this is the neutral word. Something *perdures* iff it persists by having different temporal parts, or stages, at different times, though no one part of it is wholly present at more than one time; whereas it *endures* iff it persists by being wholly present at more than one time. (Lewis 1986, p. 202)

The concepts of location are well geared to making these definitions more precise. Lewis's "exists at" is my "weakly located in"; so an object persists iff it is temporally weakly located in many, disjoint, times. The proviso "disjoint" is needed here, because if an object exists for only an instant some time in 1973, then it is weakly located in 1973, and in the 20th century, the 2nd millennium and so on. That's not the kind of multiple location we wanted. This is not a counterexample to my claim that "exists at" means weak temporal location — most likely Lewis was tacitly restricting himself to instantaneous times in the quote above, which explains why he didn't need the proviso. It would certainly not help to identify "exists at" with exact temporal location, as no object is multiply exactly located.

Though temporal location has the same formal structure as spatial location, it is not the same relation. This may seem obvious, if you think that times and places are very different kinds of entity; but I don't wish to assume that. Imagine that someone other than John Locke happened to be born in the same instant, and die in the same instant as Locke himself. Locke and this other person, then, have the same temporal exact location, but not the same spatial exact location. To avoid confusion, then, I'll apply a superscripted T to location predicates, where I intend them to have a distinctively temporal meaning.

So, to give a formal version of Lewis's definition of "persistence":

(8) $$x \text{ persists} \equiv_{df} (\exists r)(\exists s)(r \wr s \wedge x@^{T}_{\circ} r \wedge x@^{T}_{\circ} s)$$

There are two ways to formally define "perdures". One, which is suggested by the first clause of Lewis's definition, is in terms of the idea of a temporal part. This is slightly more complicated, so I will take the other route, and return to defining "temporal part" later. The second clause of Lewis's definition (and the only clause of his definition of "endures") suggests another way, in terms of "wholly located", which I defined in section 4. A thing perdures iff it persists, and no part of it is wholly temporally located at two, disjoint, times. As before, we need the proviso "disjoint" because every object is wholly located both at its exact location, and all super-regions of its exact location:

(9) x perdures $\equiv_{df}$ x persists $\wedge (\forall y)(y < x \rightarrow (\forall r)(\forall s)(x@^{T}_{\blacktriangleleft} r \wedge x@^{T}_{\blacktriangleleft} s \rightarrow r \circ s))$

Now back to temporal parts. A perduring object has different temporal parts at different times. But just what is a temporal part? We actually have a three-place relation here — a relation between an object, a time, and its temporal part *at that time*. There are pitfalls that are often ignored in discussions of how to define this: what if we allow (as I do) times to include intervals? Or arbitrary fusions of intervals?

The best way to sort this out, I think, is to start with a spatial analogy. Though analogies between time and space are controversial, they are less so among perdurantists, so using such an analogy to figure out the content of perdurantism should be unobjectionable.

When I reach my left arm out of the window of my office, there's a part of me that's in the street. Which part? My left arm. But of course if my left arm is in the street, then I have a lot of parts in the street: my left index finger, the fusion of all my left fingernails, and so on. When we say "the part of me in the street", we must mean something like "the fusion of the parts of me in the street". But that's not quite it, because I am a part of me, and I am in the street; so I, not my left arm, am the fusion of the parts of me in the street.

There are two ways to go here. We could say that my part in the street is the fusion of all of the parts of me that are wholly in the street; alternatively, we could say that my part in the street is the fusion of all of the parts of me that are entirely in the street.

To write the two definitions of temporal part down formally, we need a formal notation for mereological fusion. Following Simons (1987), I use a definite-description-like operator:

$$(\sigma x)(\Phi x) \qquad \text{for "the fusion of everything } \Phi\text{"}$$

The two definitions of temporal part are given below. I am going to argue that it is the former that is the more useful notion; but since it will be helpful to have a label for the latter, I will call it "temporal part*". There is no issue of metaphysics (or even of conceptual analysis) at stake here. "Temporal part" is a technical term of metaphysics, and I'm simply defending my choice of stipulative definition.

(10) x's temporal part at $r =_{df} (\sigma y)(y < x \wedge y@^{T}_{\blacktriangleleft} r)$

(11) x's temporal part* at $r =_{df} (\sigma y)(y < x \wedge y@^{T}_{<} r)$

Unlike some definitions of temporal part, mine do not assume that time is made up of instants — c.f. Sider (2001, p. 59–60). For all I've said, times might be gunky, every time having yet briefer times as parts, and never bottoming out in instants.

The difference between the two is how they behave when applied to enduring objects. If an object endures throughout its life, then its temporal part at each time it exists is the enduring object itself; this is because the object itself is wholly temporally located at each of the times it exists. But it has no part entirely temporally located at any subregion of the time at which it is exactly temporally located. So at some times at which it exists it has no temporal part*.

People sometimes think that temporal part* must be the concept at work when enduring objects are described as having "no temporal parts". That, however, is a mistake. It would be more consistent with the usual usage of mereological vocabulary to regard this as a typical shorthand for "no temporal *proper* parts", where a temporal proper part of an object is a temporal part (in my sense) which is also a proper part of that object. The notion of temporal part I prefer also makes it easier to give analyses of various time-indexed concepts that are neutral between enduring and perduring objects. And, as we will see, it makes it easier to make sense of the first clause of Lewis's definition of perdurance in terms of temporal parts.

Suppose we said that an object perdures iff it has different temporal parts at different times, as the first clause of Lewis's definition suggests. That definition of "perdures" would not be quite equivalent to the one in terms of "wholly located".

They come apart, however, only in fairly exotic cases. Suppose that there's something that's made up of two parts, one of which perdures in the classical way, being wholly located at no time longer than an instant, the other of which has no proper parts at all. Suppose, for example, that immanent universals, such as *humanity*, have no proper parts; but that persons perdure in the classical way. Suppose that John Locke is the only instance of *humanity*; and call the fusion of Locke with *humanity*, Locke+. Locke+ is the sort of exotic object that makes our two definitions of perdurance come apart. Locke+ has a part which is wholly located at more than one disjoint time (namely, *humanity*), so Locke+ endures by the definition in terms of "wholly located". But Locke+ also has distinct temporal parts at each time, because every temporal part of Locke+ overlaps a different temporal part of Locke, so Locke+ perdures by the definition in terms of temporal parts.

It wouldn't help with this problem to replace "temporal part" in our definition with "temporal part*". That still makes Locke+ a perdurer by one definition, and an endurer by the other. Locke+'s temporal part* at each time is identical with Locke's temporal part at that time (except for times at which Locke+ is entirely located), so Locke+ again comes out having distinct temporal parts* at each time.

This shows that it would be better to understand "different" as it occurs in both places in Lewis's first clause as "disjoint". That will bring the two definitions into line, for though Locke+ has distinct temporal parts at each time, those temporal parts are not disjoint: they overlap on *humanity* — and this is why Locke+ does not perdure. So our Lewis-inspired definition of perdurance in terms of temporal parts should read:

(12) x perdures $\equiv_{df}$ x persists $\wedge$
$(\forall r)(\forall s)(x@^T_\circ r \wedge x@^T_\circ s \wedge r \wr s \rightarrow x$'s temporal part at $r \wr x$'s temporal part at $s)$

An enduring object, according to Lewis, is one that is wholly located at more than one (disjoint) time. That is, any persisting object that does not perdure endures:

(13) x endures $\equiv_{df} (\exists r)(\exists s)(r \wr s \wedge x@^T_\blacktriangleleft r \wedge x@^T_\blacktriangleleft s)$

This means that exotic objects like Locke+ would count as enduring. Also, objects like Johnston's (1987, pp. 123–125) partial endurers — which mostly persist by enduring, but survive substantial changes only by perduring — would count as enduring by these standards. It would be useful to have a stronger notion of endurance that would require that an enduring object be wholly located at *every* time at which it exists:[9]

(14) x endures throughout $\equiv_{df}$ x persists $\wedge (\forall r)(x@^{T}_{\circ} r \rightarrow x@^{T}_{\blacktriangleleft} r)$

What we've seen here is that we can get to the allegedly technical (and sometimes, allegedly incomprehensible) concepts of temporal part, perdurance, and so on by ratcheting up from mereological relations, subregion relations among times, and the concept of exact temporal location (or alternatively, weak temporal location).

My definitions provide a good answer to those endurantists who claim that, though they can understand classical mereology, they cannot understand what a temporal part would be. Moreover, since my definitions, and the theory of location as a whole, do not decide the question of endurantism versus perdurantism — and rightly so, I think, for it is not a question to be decided on conceptual grounds alone — those endurantists cannot claim that the question has been begged against them.

Some self-ascribing endurantists, however, have claimed that my theory of location begs the question against them. Moreover, though they might welcome the clarity of my terminology of location, they would also claim that I have mischaracterised endurantism. According to these people, an enduring object is not one that is wholly located at each of many disjoint times, but one that is exactly located at each of many disjoint times. On this view, endurantism is not the denial of the temporal version of Arbitrary Partition, but of Functionality.

There are a number of things wrong with this rival interpretation of endurantism. First, there is my direct argument for Functionality: if someone doesn't believe Functionality is true, I begin to suspect they aren't talking about exact location — what part of "exact" do they not understand? There are a lot of other location relations they could be talking about instead, for example, the relation that holds between a thing x and a time t iff x is wholly and pervasively temporally located at t. That relation need not be a function, and indeed cases where it fails to be so are precisely cases in which x endures (in my sense).

Second, what unifies endurantists, on any interpretation, is their opposition to temporal parts. But the truth or falsity of Functionality has nothing to do with

[9]It still turns out that exotic objects that have some parts not in time at all do not endure throughout, even if those parts of them that are in time do.

whether objects have temporal parts. If a thing could be exactly located at two disjoint times, t and t', it could do so and have a part exactly located at t but not t', and another part exactly located at t' and not t. On the proposed interpretation of endurance, the endurantist is not denying that objects have temporal parts. This is particularly bad because perdurantists, at least, will all agree with my characterisation of *perdurantism*. If the endurantist has nothing to say that is incompatible with it, then there is no debate.

A good way of illustrating this, I think, is to refer to Cody Gilmore's (2006), which while otherwise excellent, embodies just the kind of confusion I am discussing here. Gilmore gives an informal theory of location, the location primitive of which he glosses both as "wholly present" and "exactly occupies". In his terminology, an "enduring" object "exactly occupies" many regions of space-time, while a "perduring" object "exactly occupies" just one. This terminology could be brought into line with my own if "exactly occupies" were replaced by "is wholly and pervasively located at". But it is clear that Gilmore does not want that.[10]

It might seem tempting to interpret Gilmore's "exactly occupies" as my "is exactly located at", and treat him as a denier of Functionality. That, however, would be a mistake, for Gilmore has another relation between regions and objects which, though officially defined in terms of "is exactly located at", is plainly my notion of exact location. r is the path of x, he says, iff r "exactly contains x's complete career or life-history". That is what *I* mean by "exact location", and Gilmore does not deny that no object has more than one path, which is what it would take for him to deny Functionality. I think that this will turn out to be the case for anyone who seriously tries to pursue the view that enduring objects are multiply exactly located.

Gilmore's theory of location has the first problem I mentioned: it is more charitable to regard him as affirming Functionality than as denying it. It also has the second problem. Because Gilmore's "exactly occupies" is not explained in terms of location and mereology — and indeed cannot be — it would make the question of "endurantism" vs "perdurantism" independent of the question of whether objects are divisible into arbitrary temporal parts. For all he has said, endurantism is compatible with everything advanced under the name "perdurantism" by Lewis, Sider, Quine, Smart, and so on. Though Gilmore's treatment of endurance perhaps characterises what some (in my view, confused) endurantists would like to

[10]The reasons for this have to do with the details of Gilmore's argument. If Gilmore's "exactly occupies" were my "is wholly and pervasively located at", then his examples of Cell and Tubman would "exactly occupy" all and only the same regions of space-time (whether or not they endure or perdure). It is crucial to Gilmore that this not be the case.

affirm, it is mischaracterises what they would like to deny.

6 The formal theory of location

The time has come for some details. In this section I outline the formal nature of the theory of location, and contrast the two sets of definitions I gave in section 2. The theory of location is built on a formal mereology. My mereological notation is as follows:[11]

$x \ll y$	for "x is a proper part of y"
$x < y$	for "x is a part, proper or improper, of y"
$x \circ y$	for "x overlaps (i.e. has a part in common with) y"
$x \wr y$	for "x is disjoint from (i.e. does not overlap) y"
$(\sigma x)(\Phi x)$	for "the fusion of all that is Φ"

It's useful to distinguish a reflexive from an anti-reflexive sense of "part". Traditionally, the former is called "part", the latter "proper part", each thing being its own improper part. Either can be defined in terms of the other:

(15) $$x \ll y \equiv_{df} x < y \wedge \neg x = y$$

(16) $$x < y \equiv_{df} x \ll y \vee x = y$$

Overlap and disjointness can be defined so:

(17) $$x \circ y \equiv_{df} (\exists z)(z < x \wedge z < y)$$

(18) $$x \wr y \equiv_{df} \neg(\exists z)(z < x \wedge z < y)$$

A famous feature of formal mereologies is the notion of fusion. We may speak of the fusion of all the rabbits — intuitively, the thing that's made up of,

[11] I assume the classical first order predicate calculus with identity and definite descriptions. For clarity of notation, I use the convention that, where scope is left ambiguous, unary operators have narrower scope than binary operators, and conjunction and disjunction have narrower scope than implication. Also, where I use a sentence containing an unbound variable, that variable should be treated as if bound to a universal quantifier having the widest possible scope.

and exhausted by, all the rabbits. This is usually written formally as a definite-description-like operator.[12]

$$(\sigma x)(\Phi x) =_{df} (\iota x)(\forall y)(x \circ y \leftrightarrow (\exists z)(\Phi z \wedge y \circ z)) \quad (19)$$

There's a common misapprehension about the concept of mereological fusion. People sometimes think of mereological fusions as being a distinctive class of entities, with, perhaps, their own distinctive range of essential properties. This way of thinking is perhaps suggested by an analogy with set theory, with which classical mereology is sometimes compared. Nothing could be further from the truth. Mereological fusion means nothing more nor less than what's said in the definition given above, and that definition is stated in a non-modal language. The mereological fusion of the Φs is just whatever it is that overlaps everything that overlaps a Φ. If that thing — whatever it is that overlaps everything that overlaps a Φ — is a bicycle, then it has whatever essential properties are appropriate to bicycles; if it is a hedgehog, then it has whatever essential properties are appropriate to hedgehogs. If it is something we have no name for apart from "the mereological fusion of the Φs", then for all we've said it has no essential properties at all. Someone who disagrees with this is simply speaking at cross-purposes with me: they have a stronger definition of "fusion" than that given by formal mereology.

Finally, for the mereological part of the theory of location, we need some mereological axioms. The following give us Simons' (1987, pp. 25–31) *Minimal Extensional Mereology* (MEM):

(Asymmetry)	$x \ll y \rightarrow \neg y \ll x$
(Transitivity)	$x \ll y \wedge y \ll z \rightarrow x \ll z$
(Weak supplementation)	$x \ll y \rightarrow (\exists z)(z \ll y \wedge z \wr x)$
(Maximal common part)	$x \circ y \rightarrow (\exists z)(\forall w)(w < z \leftrightarrow w < x \wedge w < y)$

There's another famous principle of mereology that's worth mentioning here, though it is *not* a theorem of MEM. This is the principle of unrestricted fusion. If it were added to the system, we would have Classical Extensional Mereology (CEM), which is strictly stronger than MEM:

(Unrestricted fusion) $(\exists x)(\Phi x) \rightarrow (\exists x)(\forall y)(x \circ y \leftrightarrow (\exists z)(\Phi z \wedge y \circ z))$

[12] The definition of fusion given here is, strictly speaking, a definition schema.

To MEM we add the four location relations already described:

$x@_\circ r$	for "x is weakly located at r"
$x@_< r$	for "x is entirely located at r"
$x@_> r$	for "x is pervades r"
$x@r$	for "x is exactly located at r"

There are two equivalent ways of defining these. We might make @ primitive, with functionality as an axiom. Call the system that includes MEM, the definitions (1), (2), and (3), and the functionality axiom, $S@$:

(1) $$x@_< r \equiv_{df} (\exists s)(x@s \wedge s < r)$$

(2) $$x@_> r \equiv_{df} (\exists s)(x@s \wedge r < s)$$

(3) $$x@_\circ r \equiv_{df} (\exists s)(x@s \wedge r \circ s)$$

(Functionality) $$(x@r \wedge x@s) \rightarrow r = s$$

The alternative is to have $@_\circ$ primitive, and exactness as an axiom. Call the system that includes MEM, the definitions (4), (5), and (6), and the exactness axiom, $S@_\circ$:

(4) $$x@_< r \equiv_{df} x@_\circ r \wedge (\forall s)(x@_\circ s \rightarrow r \circ s)$$

(5) $$x@_> r \equiv_{df} (\forall s)(r \circ s \rightarrow x@_\circ s)$$

(6) $$x@r \equiv_{df} (\forall s)(r \circ s \leftrightarrow x@_\circ s)$$

(Exactness) $$(\exists r)(x@_\circ r) \rightarrow (\exists r)(x@r)$$

Both of these systems affirm exactness and functionality. In $S@$, exactness is a consequence of (3). In $S@_\circ$, functionality is a consequence of (6) and the extensional overlap principle (20), which is itself a theorem of MEM:

(20) $$x = y \leftrightarrow (\forall z)(x \circ z \leftrightarrow y \circ z)$$

To see that exactness is independent of MEM plus the definitions of $S@_\circ$, consider the following counter-model: there are only three places, r, s, t, and one thing, a, all mereologically atomic. a is weakly located at r and s, but not at t. In this model, *a* has no exact location. This is not, of course, a model of CEM, which does not allow "flat" models like this one. Exactness would follow from (6) with the unrestricted fusion principle.

The definitions of $S@$ are also derivable in $S@_\circ$, in the sense that rules that would allow you to replace $@_\circ$, $@_<$ and $@_>$ with their definitions in $S@$ are derivable in $S@_\circ$. Similarly, the definitions of $S@_\circ$ are derivable in $S@$. In fact, the systems are equivalent.

7 Casati and Varzi

I'm not the first person to try to say something about the formal structure of location relations. It might help to contrast my proposals with the system described by Casati and Varzi (1999) Their theory of location is based on a formal mereotopology they call GEMTC. For ease of presentation, I'll ignore the topological features of this theory, and describe it as it might be set up on the basis of a pure mereology. To this, Casati and Varzi add a primitive predicate of exact location. Like me, they treat the subregion relation among places and the part-whole relation among things in space as one mereological relation, and they define a number of inexact location relations:

(Partial Location)	$PL(x,r) \equiv_{df} (\exists z)(z < x \wedge z@r)$
(Whole Location)	$WL(x,r) \equiv_{df} (\exists s)(s < r \wedge x@s)$
(Generic Location)	$GL(x,r) \equiv_{df} (\exists z)(\exists s)(z < x \wedge s < r \wedge z@s)$

Casati and Varzi's "whole location" is my entire location. Their "partial location" and "generic location" correspond to my pervasive and weak location respectively (and this is made clear by the intuitive glosses they give on these concepts) but are defined differently.

They give the following axioms, intended to capture the formal features of location:

(Functionality)	$(x@r \wedge x@s) \rightarrow r = s$
(Conditional reflexivity)	$x@r \rightarrow r@r$
(Weak expansivity)	$x < y \wedge x@r \wedge y@s \rightarrow r < s$
(Arbitrary partition)	$r < s \wedge x@s \rightarrow PL(x,r)$

Functionality we have already discussed. The other three axioms each raise interesting questions.

Conditional reflexivity raises the question of what the domain of the location relations is. I've been loosely talking of "things" located in "places". But what about those places, where are they located, if anywhere? I do not see how to assign any metaphysical significance to this question. We might decide to say that every place is exactly located at itself (this is, effectively, what Casati and Varzi are saying here); or we might decide to say that no place is located anywhere (in the meaning of "location" in use). These seem to me to be equally good stipulations. So I have no quarrel with conditional reflexivity; but nor with its denial.

Casati and Varzi's expansivity principle is weaker than the one I described in section 4. I describe the type of principle they use as "weak expansivity". Suppose that there are some objects that are outside of space and time, which have no location at all. It is consistent with weak expansivity that these objects have parts that are in space or time; perhaps even all of their proper parts. Expansivity ought to rule this out, I think, and accordingly I recommend my principle as a friendly amendment to Casati and Varzi's system.

Finally, there is Casati and Varzi's version of the arbitrary partition principle, of which, as I've intimated, I take a dim view. I think that this axiom is an unwarranted metaphysical excrescence on a formal theory of location, and would be better removed. Unfortunately, it's not easy to remove it from Casati and Varzi's system, and the reasons why it is not easy shed some light on why people might mistakenly suppose that it is a conceptual truth. The reason has to with the differences between Casati and Varzi's definitions of the inexact location predicates and my own.

Casati and Varzi's relation of Partial Location corresponds to my pervasive location; and Generic Location corresponds to my weak location. But the definitions are very different (and in the context of my system, inequivalent). To see this, begin by noting that WL and PL are not, as Casati and Varzi claim them to be, dual concepts. (Casati and Varzi 1999, p. 120) $WL(x,r)$ says there is a certain kind of place, namely x's exact location, and that r is related to it in a certain way. $PL(x,r)$, by contrast, says that there is a certain kind of *thing*, namely a part of x.

If Casati and Varzi really want the dual to WL, they should define it this way:

(21) $$PL'(x,r) =_{df} (\exists s)(r < s \wedge x@s)$$

This PL' relation is what I've called "pervasive location". Once we see the distinction between PL and PL', it's easy to see why Casati and Varzi want the arbitrary partition axiom. It erases that distinction, for in their system, it turns out, $PL'(x,r) \leftrightarrow PL(x,r)$ is a theorem, and Arbitrary Partition is needed to get the left to right conditional.

Without Arbitrary Partition, the definitions of *PL* and *GL* would be too strong to capture the intuitive meaning that Casati and Varzi's glosses make it clear that they are supposed to have. Suppose that I stand in the doorway of my office, one foot on the floor of the office, another foot on the floor of the corridor. I want to say that I am inexactly located in my office, in some sense. Here's a good sense in which I am: my exact location and my office overlap; they have an intersection — a region which is part of both. But, absent Arbitrary Partition, that doesn't suffice for me being inexactly in my office in any of Casati and Varzi's senses. They require that in addition to the intersection existing, there is some part of me that exactly occupies the intersection.

This requirement is gratuitous: if we used PL' instead of *PL*, and made similar revisions to the definition of *GL*, it would go away. The revised definitions would capture the intuitive meaning *PL* and *GL* are supposed to have without the need for Arbitrary Partition, rendering Arbitrary Partition (quite properly) an independent extra that might be affirmed or denied according to metaphysical taste. Besides that, the revised definitions would better capture the formal relationship between PL and WL. This is just what I've done in my definitions of "pervades" and "entirely located".

8 Things and places

The theory of location has a striking *prima facie* ontological commitment: it is substantivalist, and in this it reflects the ordinary way we talk about places and times. It also reflects the way philosophers of time have talked about persistence in terms of "existing at" multiple times. Though we talk this way, however, many of us are worried about the ontological commitment to places and times — are there really such things? If so, what are they like? If not, how can we understand talk that is apparently about them?

These questions are particularly pressing when we use the theory of location as a way of expressing theories about temporal parts. Participants in the temporal parts debate do not take themselves to be exploring a number of variants of substantivalism; rather, they are asking questions that should be, for the most part independent of substantivalism.

There are three general answers to the problem posed by apparent commitment to substantivalism. First up, we could go for a form of *relationalism*: say that the formalism of the theory of location is metaphysically misleading, precisely because of its commitment to times and places. But then we have a problem - how do we understand discussions of temporal parts, of endurance, or “wholly located”? Perhaps such talk could be paraphrased into a suitable relationalist framework. I have some hope of doing this myself, but it would be a difficult task, and beyond the scope of this paper.

The second option I call *anti-reductionist substantivalism*. On this view, we take the seeming commitments of the theory of location with full metaphysical seriousness. There are fundamentally at least two kinds of object in the world, material ones (things) and immaterial ones (places); and there is a fundamental and unanalysable external relation of location (exact or weak) between them. Nothing can be located at a thing; perhaps, with Casati and Varzi, we should say that each place is exactly located at itself; perhaps that no place can be located anywhere.

There might also be mixed objects — mereological fusions of things and places — some of whose parts are material and some immaterial. These seem like odd creatures. Where should we say that they are located? If each place is exactly located at itself, and strong expansivity is true, then a fusion of a thing x with a place r should at least pervade the fusion of r with the exact location of x. Perhaps we should say that mixed objects are always exactly located at the fusion of the exact locations of their parts. Though this answer is not arbitrary, it does not exactly help us understand what a mixed object might be like. Perhaps we should say that there cannot be mixed objects. That would be compatible with all I have said above; but it would not be compatible with a principle of unrestricted mereological fusion, which many people who would like to use the theory of location may be attracted to.

Here is another problem. Suppose a material object, x, is cubical. We are accustomed to thinking that shape properties are intrinsic. But surely, whether x is cubical depends on what sort of place it is exactly located in. A cubical object can’t be located at a non-cubical place; but now we have a necessary connection between the intrinsic nature of x and the external relations it bears to places, and indeed between the intrinsic nature of x and the intrinsic nature of its exact location. That should be impossible — it is a violation of a necessary condition on the set of intrinsic properties. (Weatherson 2001, pp. 369–373)

A good way to solve both these problems at one blow is to say that ordinary material objects, like chairs and tables, and you and me, *are* mixed objects; specifically, that we have our exact locations as proper parts. That way, *being cubical* is like having a nose — it is an intrinsic relational property[13] — and the necessary condition on the set of intrinsic properties is not violated because there is only a necessary connection between the intrinsic nature of x and the external relations it bears to some of its own parts — to things that are not wholly distinct from it.

This leaves us with another difficulty. What about the purely material part of an ordinary cube? On our current proposal, there must be such a thing; and it cannot be cubical in any ordinary sense. Where is it located? Perhaps it is not in space at all — that would be weird. But if it is in space, then where? Is it exactly located just where the ordinary cube is? If so, it would seem to exactly occupy a cubical place without itself being cubical.

The third possibility is *reductionist substantivalism*. I have taken care to ensure that the theory of location is compatible with a striking and surprising thesis: that each thing is exactly located at itself and nowhere else. This might be called the "identity theory of location":[14]

(Identity Theory of Location) $$x@r \leftrightarrow x = r$$

The typical metaphysical development of this view holds that places are much as the anti-reductionist substantivalist thinks they are — parts of a (mostly) immaterial plenum filling the entire universe — material things, however, are boldly identified with "matter filled" places. "Matter filled", here, however, must not be understood to express a relation to some piece of matter that fills a place. Rather, it is an intrinsic property of some places that they are matter-filled. One feels rather as though this view is an eliminative reduction of matter, though its advocates might not see it that way.

The drawback of this is that the principle of arbitrary partition follows from the identity theory. (A proof of this is sketched in the appendix). So, if you are not

[13]Intrinsic properties are ones which "concern how their instances are in themselves" and can be contrasted with extrinsic properties. Relational properties are those that in some sense "involve a relation". In this terminology, due to Lloyd Humberstone (1996), relational properties may be intrinsic or extrinsic.

[14]John Hawthorne suggested an interesting variant on the identity theory of location to me — the "coincidence theory" of location. The coincidence theory would hold that x is exactly located at r iff x overlaps all and only the same things that r does. In the context of MEM, this is equivalent to the identity theory, but in a weaker, non-extensional mereology, it would not be. Since any two coinciding things must have all and only the same parts, we would still have Arbitrary Partition as a consequence. However, the coincidence theory could be held by a anti-reductionist substantivalist, since it allows for things to be distinct from their locations.

attracted to the principle of arbitrary partition, you will not want to be a reductionist substantivalist. Sider (2001, pp. 110–119) has used this type of argument as an argument *for* arbitrary partition, at least in the temporal case. We should be substantivalists, he thinks, and substantivalists should be reductionists, or risk miring themselves in the sorts of difficulties described above. But reductionistic substantivalism entails arbitrary partition, so we should believe in arbitrary partition.[15] It would go beyond the scope of this paper to pass judgment on this argument here, but I think that it is worthy of attention.

Appendix

In this section I present some theorems of the theory of location alluded to in the text. The proofs are given in a slightly abbreviated natural deduction form. They are intended as examples of how to formally reconstruct some of the arguments given in the text, rather than being of any logical interest.

Functionality is a theorem of $S@_\circ$ The functionality principle is an axiom of $S@$. Showing that it is a theorem of $S@_\circ$ is part of what's needed to show that the two systems are equivalent, as discussed in section 6.

(22)	$x@r \wedge x@s$	(assumption for conditional proof)
(23)	$(\forall t)(x@_\circ t \leftrightarrow t \circ r)$	(22, ∧-elim, definition of @)
(24)	$(\forall t)(x@_\circ t \leftrightarrow t \circ s)$	(22, ∧-elim, definition of @)
(25)	$(\forall t)(t \circ r \leftrightarrow t \circ s)$	(23, 24)
(26)	$(\forall t)(t \circ r \leftrightarrow t \circ s) \rightarrow r = s$	(theorem of extensional mereology)
(27)	$r = s$	(25, 26, modus ponens)
(28)	$x@r \wedge x@s \rightarrow r = s$	(27, conditional proof, discharging 22)

Adequacy of $S@$**-definitions in** $S@_\circ$ The location predicates are defined differently, and in terms of a different primitive in $S@$ and $S@_\circ$. However, they are, in an important sense, synonymous. One way to formally show this is to show that the definitions of each system are derived rules of the other. It would be a long and boring task to do this for each definition. Instead I will give a couple of illustrative examples.

[15] The argument for substantivalism as against relationalism cited here (though not the argument for reductionist as against anti-reductionist substantivalism) is given in greater detail in Hawthorne and Sider (2002).

I start by deriving the definition of $@_\circ$ in terms of $@$ in $S@_\circ$. That is, I am going to show that $x@_\circ r$ and $(\exists s)(x@s \wedge r \circ s)$ are inter-derivable in $S@_\circ$. The other definitions of $S@$ can be derived in $S@_\circ$ in a similar way. First, the right to left direction, which is very easy:

(29)	$(\exists s)(x@s \wedge r \circ s)$	
(30)	$(\exists s)((\forall t)(x@_\circ t \leftrightarrow t \circ s) \wedge r \circ s)$	(29, definition of @)
(31)	$(\forall t)(x@_\circ t \leftrightarrow t \circ s) \wedge r \circ s$	(30, ∃-elimination)
(32)	$(\forall t)(x@_\circ t \leftrightarrow t \circ s)$	(31, ∧-elimination)
(33)	$x@_\circ r \leftrightarrow r \circ s$	(32, ∀-elimination)
(34)	$r \circ s$	(31, ∧-elimination)
(35)	$x@_\circ r$	(33, 34, modus ponens)

Now the left to right direction. To do this we need to appeal to the exactness axiom of $S@_\circ$:

(36)	$x@_\circ r$	
(37)	$(\exists s)(x@_\circ s) \rightarrow (\exists s)(\forall t)(x@_\circ t \leftrightarrow t \circ s)$	(exactness axiom)
(38)	$(\exists s)(x@_\circ s)$	(36, ∃-introduction)
(39)	$(\exists s)(\forall t)(x@_\circ t \leftrightarrow t \circ s)$	(37, 38, modus ponens)
(40)	$(\forall t)(x@_\circ t \leftrightarrow t \circ s)$	(39, ∃-elimination)
(41)	$x@_\circ r \leftrightarrow r \circ s$	(40, ∀-elimination)
(42)	$r \circ s$	(36, 41, modus ponens)
(43)	$(\forall t)(x@_\circ t \leftrightarrow t \circ s) \wedge r \circ s$	(40, 42, ∧-introduction)
(44)	$(\exists s)((\forall t)(x@_\circ t \leftrightarrow t \circ s) \wedge r \circ s)$	(43, ∃-introduction)
(45)	$(\exists s)(x@s \wedge r \circ s)$	(44, definition of @)

So, in $S@_\circ$:

(46) $$x@_\circ r \dashv\vdash (\exists s)(x@s \wedge r \circ s)$$

Adequacy of $S@_\circ$-definitions in $S@$ Similarly, it would be nice to derive the definitions of $S@_\circ$ in $S@$. My example will be the definition of @ — I will show that $x@r$ and $(\forall s)(r \circ s \leftrightarrow x@_\circ s)$ are inter-derivable. This proof appeals to the functionality axiom of $S@$, and to some theorems of mereology. First, the left to right direction:

(47)	$x@r$	
(48)	$r \circ s$	(assumption)
(49)	$(\exists t)(x@t \wedge t \circ s)$	(47, 48, $\exists$-introduction)
(50)	$r \circ s \rightarrow (\exists t)(x@t \wedge t \circ s)$	(49, conditional proof, discharging 48)
(51)	$(\exists t)(x@t \wedge t \circ s)$	(assumption)
(52)	$x@t \wedge t \circ s$	(51, $\exists$-elimination)
(53)	$x@r \wedge x@t \rightarrow r = t$	(functionality axiom)
(54)	$r = t$	(52, 53, modus ponens)
(55)	$r \circ s$	(52, 54, $\wedge$-elimination, substitution)
(56)	$(\exists t)(x@t \wedge t \circ s) \rightarrow r \circ s$	(55, conditional proof, discharging 51)
(57)	$(\forall s)(r \circ s \leftrightarrow (\exists t)(x@t \wedge t \circ s))$	(50, 56, $\forall$-introduction, $\leftrightarrow$-introduction)
(58)	$(\forall s)(r \circ s \leftrightarrow x@_{\circ}s)$	(57, definition of $@_{\circ}$)
(59)	$x@r \vdash (\forall s)(r \circ s \leftrightarrow x@_{\circ}s)$	

And now the right to left direction. This is more difficult, so I have annotated the proof in more detail. Our first task is to show that (60) entails that *x* has an exact location, and give that exact location a name, *u*:

(60)	$(\forall s)(r \circ s \leftrightarrow x@_{\circ}s)$	
(61)	$(\forall s)(r \circ s \leftrightarrow (\exists t)(x@t \wedge t \circ s))$	(60, definition of $@_{\circ}$)
(62)	$r \circ r \leftrightarrow (\exists t)(x@t \wedge t \circ r)$	(61, $\forall$-elimination)
(63)	$r \circ r$	(theorem of extensional mereology)
(64)	$(\exists t)(x@t \wedge t \circ r)$	(62, 63, modus ponens)
(65)	$x@u$	(64), $\exists$-elimination, $\wedge$-elimination)

Now we prove that *u* is identical to *r*, using the extensional overlap principle (20) mentioned in section 6. First we show that if *u* overlaps an arbitrary *s*, then *r* does too:

(66)	$u \circ s$	(assumption)
(67)	$x@u \wedge u \circ s$	(65, 66)
(68)	$(\exists t)(x@t \wedge t \circ s)$	(67, $\exists$-introduction)
(69)	$r \circ s \leftrightarrow (\exists t)(x@t \wedge t \circ s)$	(61, $\forall$-elimination)
(70)	$r \circ s$	(68, 69, modus ponens)
(71)	$u \circ s \rightarrow r \circ s$	(70, conditional proof discharging 66)

Now we show that if r overlaps an arbitrary s, so does u:

(72)	$r \circ s$	(assumption)
(73)	$(\exists t)(x@t \wedge t \circ s)$	(72, 69, modus ponens)
(74)	$x@t \wedge t \circ s$	(73, $\exists$-elimination)
(75)	$x@u \wedge x@t \rightarrow u = t$	(functionality axiom)
(76)	$u = t$	(65, 74, 75)
(77)	$u \circ s$	(74, 76, substitution, $\wedge$-elimination)
(78)	$r \circ s \rightarrow u \circ s$	(77, conditional proof discharging 72)

But then, r and u overlap all and only the same things, and so are identical, by the extensional overlap principle. So, since x is exactly located at u, x is exactly located at r:

(79)	$r \circ s \leftrightarrow u \circ s$	(71, 78)
(80)	$(\forall s)(r \circ s \leftrightarrow u \circ s)$	(79, $\forall$-introduction)
(81)	$r = u \leftrightarrow (\forall s)(r \circ s \leftrightarrow u \circ s)$	(theorem of extensional mereology)
(82)	$r = u$	(80, 81)
(83)	$x@r$	(65, 82)

So, in $S@$:

(84) $$x@r \dashv\vdash (\forall s)(r \circ s \leftrightarrow x@_{\circ}s)$$

The other definitions of $S@_{\circ}$ can be derived in a similar way.

Arbitrary partition is entailed by the Identity Theory of Location In section 8 I claimed that reductionistic substantivalism entails the arbitrary partition principle, via the "Identity Theory of Location" — the view that each thing is exactly located only at itself.

(85)	$x@_{>}r$	(assumption for conditional proof)
(86)	$(\exists s)(x@s \wedge r < s)$	(85, definition of $@_{>}$)
(87)	$x@s \wedge r < s$	(86, $\exists$-elimination)
(88)	$x@r \leftrightarrow x = r$	(identity theory of location)
(89)	$x = s$	(87, 88, modus ponens)
(90)	$r < x$	(87,89, $\wedge$-elimination, substitution)
(91)	$r = r$	($=$-introduction)
(92)	$r@r$	(91, 88, modus ponens)
(93)	$(\exists y)(y < x \wedge y@r)$	(90, 92, $\exists$-introduction, $\wedge$-introduction)
(94)	$x@_{>}r \rightarrow (\exists y)(y < x \wedge y@r)$	(93, conditional proof, discharging 85)

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